NUMERICAL ASPECTS OF BUBBLE NUCLEATION

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Bubble nucleation has been studied on lattices using phenomenological Langevin equations. Recently there have been theoretical motivations for using these equations. These studies also conclude that the simple Langevin description requires some modification. We study bubble nucleation on a lattice and determine effects of the modified Langevin equations.

1. Introduction

The effective potential determines the vacuum state of a quantum field theory.[1] When radiative corrections are included or, if one constructs the finite temperature version of field theory the (effective)potential may have two minimum, one of which is lower than the other. The 'higher' minimum is referred to as the false or metastable vacuum. The global minimum is the true vacuum state of the theory. The existence of these metastable vacua have interesting consequences in the early Universe. If at early times the field settles in the false vacuum it may 'decay' to the true vacuum either by quantum tunneling or thermal hopping. The transition region is spherically symmetric in coordinate space. Bubbles of the stable phase appear in the metastable phase. In relativistic field theory a formalsim for calculating decay rates from the unstable to the stable vacuum has been developed.[2] Since potentials which are non-linear in the fields are inherent in these problems, it is usually not always possible to find analytic solutions to the decay rate equations. Numerical methods are employed in order to avoid too many analytic approximations. For example, in one detailed numerical study, a scalar field in $1 + 1$ dimensions in contact with a thermal bath was modeled using a phenomenological Langevin equation with a field independent, white noise driving term.[3] There have been attempts to derive a Langevin equation containing fluctuation and dissipation terms from purely field theory considerations.[4] In these works the authors considered different models for the thermal bath, but the common result obtained was the possiblity that the thermal noise in the Langevin equation could be colored and depend on the field. In this work we investigate the effects of these non-linear fluctuation and dissipation terms numerically.

2. Numerical Method

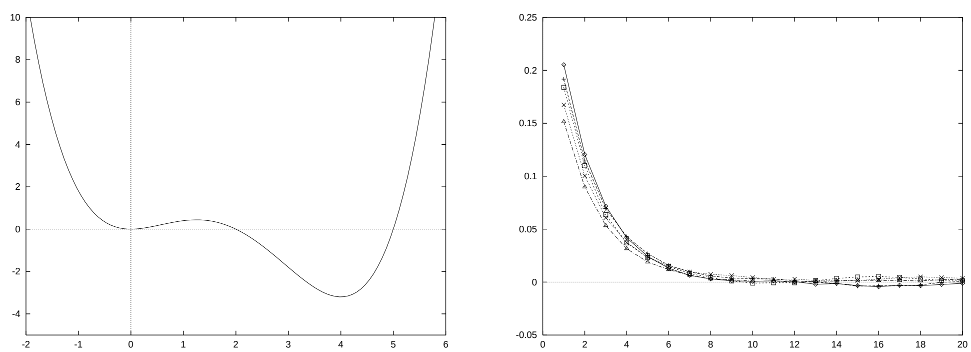
We study nucleation by evolving the equations of motion of the field on a lattice using a staggerd leap-frog algorithm. We work in $1+1$ dimensions and assume that the decay is thermally driven. The field is coupled to the thermal bath through a stochastic noise term, $\xi(x,t)$.

$$\frac{\partial^2\phi(x,t)}{\partial x^2} - \frac{\partial^2\phi(x,t)}{\partial t^2} - \eta F(\phi(x,t))\frac{\partial\phi(x,t)}{\partial t} = -V(\phi) + \xi(x,t)G(\phi(x,t)) \qquad (1)$$

where $F(\phi(x,t)) = G(\phi(x,t)) = 1$ for *additive noise* and, $F(\phi(x,t)) = \phi(x,t)^2$ and $G(\phi(x,t)) = \phi(x,t)$ for *multiplicative noise*. The general shape of the potential is show in Fig. 1(a). The noise is assumed to be Gaussian, in terms of the distribution of sizes of the fluctuations, and white, in that the fluctuations are uncorrelated in space and time. We assume the noise and viscosity terms are related through the fluctuation-dissipation theorem.[5]

$$< \xi(x,t)\xi(x',t') >= 2T\eta\delta(x-x')\delta(t-t'). \qquad (2)$$

In later work a generalized fluctuation-dissipation theorem for the multiplicative noise will be discussed. We start with the field in the false vacuum, $V(\phi) = 0$, and evolve the equation of motion on the lattice until the field acquires its true vacuum value at enough neighboring space points to avoid counting as bubbles fluctuations that re-collapse to the false vacuum value.

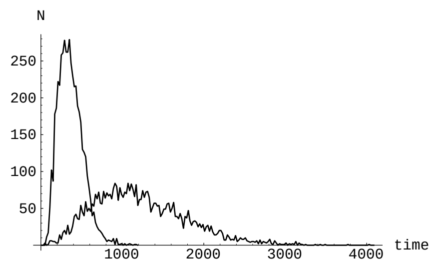


(a) (b)

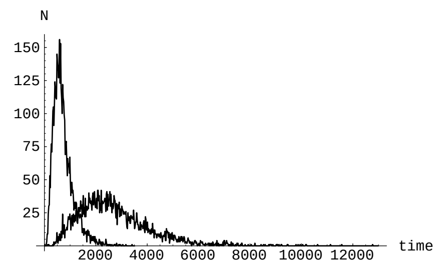
Fig. 1. (a)The potential $V(\phi) = \phi^2 - \alpha\phi^3 + \lambda\phi^4$ vs ϕ: (b)The correlation $< \phi(x)\phi(x') >$ vs $|x-x'|$ for initial conditions at different temperatures

3. Results

We report on some observations and preliminary results of our study. A large number of nucleation times has to be obtained in order to make a good estimate of the nucleation rate. Furthermore, the nucleation times obtained from the simulation have to be suitably interpreted. We found that if one starts in the false vacuum with a uniform field configuration or with one which is random and uncorrelated then there is a long waiting time before the any bubble is nucleated. This delay is interpreted as the time taken for the coupled field to acquire the short range correlation that is needed for a bubble to nucleate. Nucleation of bubbles occurs only when the field at some point in space reaches the true vacuum and *pulls* along neighbouring points. A quenching technique, where the equations are evolved in an unbroken potential until the short distance correlation, Fig.1(b), is obtained, can be used to eliminate the delay time. Fig. 2(a) and (b) show the nucleation times recorded for 5000 bubbles using separately the additive and multiplicative noise from (1). We used $\alpha = 0.74$, $\lambda = 0.1$, and $\eta = 1$. At the time of quenching



(a)



(b)

Fig. 2. Nucleation times at (a) T=1.6 and (b) T=1.1. Ther sharper peaks are the additive noise cases.

we reset the time to zero. We see that the distributions are not symmetric. This is expected for the distribution of waiting times for a decay process. We also see that the for the multiplicative noise case the nucleation times are larger and more broadly distributed. Although the size of the thermal fluctuations is multiplied by the field value, the nonlinear dissipation term has an overall larger effect and retards the decay in comparison to the additive noise case. If one obtains non-linear Langevin type equations when studying realistic models like the Electroweak effective potential, for example, the implication for the time scale of nucleation will require a similar careful study. Other aspects of these non-linear models currently under study include the effects on bubble wall expansion and the spatial distribution of critical fluctuations.

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